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Entropies and Entanglement for Initial Mixed State in the Multi-quanta JC Model with the Stark Shift and Kerr-like Medium

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In this paper, we study the time evolution of the entropies and the degree of entanglement in the mixed state for a multi-quanta JC model taking into consideration Stark shift and Kerr-like medium effect, we use a numerical method to investigate the time evolution of the partial entropy of the atom and field subsystem. This is done in the framework of the multi-quanta presses JC model with both the Stark shift and Kerr-like medium effect added. Furthermore, we examine the effect of the superposition states and a statistical mixture of coherent states as an initial field on the entropies and entanglement. Our results show that the setting of the initial state play an important role in the evolution of the sub-entropies and entanglement.

KEY WORDS: entropies; the degree of entanglement; mixed state; the superposition states and a statistical mixture of coherent states.

1. INTRODUCTION

Entanglement is one of the key properties distinguishing quantum theory from classical descriptions of the world. Naturally it has become a subject of intensive study in fundamental physics. Recently it has also been identified as a central ingredient to facilitate possible practical applications as quantum cryptography and quantum computing. As a simple quantum system to generate and observe entanglement, the dynamics of a field mode coupled to a single atom in the Jaynes-Cummings (JC) model has been extensively studied (Phoenix and Knight, 1988, 1990, 1991a,b; Buzek *et al.*, 1992; Knight and Shore, 1993; Buzek and Hladky, 1993; Fang and Zhou, 1993, 1994a,b,c; Fang, 1994a,b; Obada and Abdel-Aty, 2000; Abdel-Aty *et al.*, 2002a,b). In this context it was shown that calculating the

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partial entropies of the field S_f or the atom S_a can be used as an operational measure of the entanglement degree of the generated quantum state (Bennett *et al.*, 1996a). One finds that the higher the entropies (S_f, S_a), the greater the entanglement. Starting from an initial atom-field product state one can find perfectly entangled states between field and atom at certain later times even for initial coherent states with large photon number (Phoenix and Knight, 1988; Phoenix and Knight, 1990; 1991a,b; Buzek *et al.*, 1992). However, the time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement only if one deals with a pure state of the system with zero total entropy $S = 0$.

There has been considerable interest in the properties of the so-called superposition states of light (*SS*) involving superpositions of coherent states with strongly differing amplitude (Mandel, 1986; Wodkiewicz *et al.*, 1987; Janszky and Vinogradov, 1990; Buzek and Knight, 1991; Schleich *et al.*, 1991; Vidiella-Barranco *et al.*, 1992). One particularly interesting case is the superposition of two coherent states of fixed amplitude but opposite phase (Mandel, 1986; Janszky and Vinogradov, 1990; Buzek and Knight, 1991; Schleich *et al.*, 1991; Vidiella-Barranco *et al.*, 1992). Due to the quantum interference, the properties of such a superposition are very different from the properties of the constituent states (coherent states), as well as from the incoherent superposition or statistical mixture (*SM*) of coherent states.

In this article, we use a numerical method to measure the degree of entanglement in the mixed state. We investigate the evolution of the partial entropy of the atom and field subsystem in the frameworks of the JC model, which can give an answer to the question of quantum entanglement in the composite state. Different features of the entanglement are investigated when the field is initially assumed to be in a coherent state, an even coherent state (Schrodinger cate state) and a statistical mixture of coherent states. Furthermore, we examine the effect of the superposition states of light on the entropies and entanglement in the framework of the multi-quanta process JC model with an added both the Stark shift and Kerr-like medium. This model consist of a single two-level particle interaction with a single-mode field surrounded by a nonlinear Kerr-like medium contained inside a very good quality cavity. The cavity mode is coupled to the Kerr medium as well as to the two-level atom. The Kerr medium can be modelled as an harmonic oscillator with frequency ω (Joshi and Puri, 1992). We compare the behaviour of the system in the case of having a coherent superposition state and a statistical mixture of coherent states as an initial field. It is shown that the superposition states, Stark shift and the Kerr-like medium plays an important role in the evolution of the entropy and entanglement in the multi-quanta process JC model.

The material of this article is arranged as follows. In section II, we find the wave function for the system and write the expressions for the final state vector at any time $t > 0$. By a numerical computation, we investigate the entropies (S_a, S_f) and the particle-field entanglement with an additional both the Stark shift and the

Kerr-like medium by comparing the dynamics of *SS* and *SM* states for one-photon process and for two-photon process input in Section III. Finally, conclusion are provided.

2. QUANTUM DYNAMICS OF THE JC MODEL

The JC Hamiltonian describing the multi-quanta process interaction of a two-level particle (atom or trapped ion) interacting with a single-mode quantized field under the RWA is given by

$$\hat{H} = \frac{\omega_o}{2}\hat{\sigma}_z + \omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \chi\hat{a}^{\dagger 2}\hat{a}^2 + \hat{a}^\dagger\hat{a}(\beta_2\,|e\rangle\,\langle e| \\ +\beta_1\,|g\rangle\,\langle g|)\,\delta(k-2) + \lambda(\hat{a}^{\dagger k}\hat{\sigma}_- + \hat{a}^k\hat{\sigma}_+)\;(\hbar = 1), \tag{1}$$

where ω is the frequency of the cavity field, ω_o is the transition frequency between the excited and ground states of the particle (atom or trapped ion), $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and the creation operators of the cavity field respectively; λ is the particle-field coupling constant, $\hat{\sigma}_z$ is the population inversion operator, and $\hat{\sigma}_\pm$ are the "spin flip" operators, $\delta(k-2)$ is the delta function ($\delta(k-2) = 1$ if $k = 2$ and zero otherwise), β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transitions to the intermediate relay level, We denote by χ the dispersive part of the third-order nonlinearity of the Kerr-like medium, with the detuning parameter $\Delta = \omega_o - \omega k$. It is interesting to note that when we put $K = 1$ and $\beta_1 = \beta_2 = 0$, i.e., in the absence os Stark shift, we get the results of Ref. (Obada and Abdel-Hafez, 1986; Buck and Sukumar, 1989; Buek, 1989; Buek and Jex, 1989; Bhattacherjee and Gangopadhyay, 1996; Bhattacherjee, 1997).

Let us consider the particle in its excited state $|e\rangle$, i. e., $|\psi_a(0)\rangle = |e\rangle$, and the field is assumed to be initially in a superposition states, i. e.,

$$|\psi_{SS}(0)\rangle = \frac{1}{\sqrt{A}}\,[|\alpha\rangle + r\,|-\alpha\rangle] \tag{2}$$

where $A = [1 + r2 + 2r\exp(-2\alpha^2)]$, with α real. The parameter r can assume the values -1, 0 and 1, which corresponds to an odd coherent state, a coherent state and an even coherent state respectively. As we know, because the interference term in Eq. (2) have a rapid decay to a *SM* when we include dissipation, so we want to see how different would be the behaviour of the system if the input states are statistical mixture of the states $|\alpha\rangle$ and $|-\alpha\rangle$, i.e.,

$$\rho_{SM}(0) = \frac{1}{2}\,[|\alpha\rangle\,\langle\alpha|] + |-\alpha\rangle\,\langle-\alpha|] \tag{3}$$

with

$$|\alpha\rangle = \sum_{n=0}^{\infty} q_n \, |n\rangle = \sum_{n=0}^{\infty} e^{-|\alpha^2|/2} \frac{\alpha^n}{\sqrt{n!}} \, |n\rangle \, . \tag{4}$$

To treat the superposition state and the mixture state on a similar manner, we introduce a functional pseudo wave function in the form:

$$|\psi_{SM}(0)\rangle = \sum_{n} C_n q_n \, |n\rangle \tag{5}$$

with

$$C_n = \frac{1}{\sqrt{2}} \left[\delta_1 + (-1)^n \delta_2\right],$$

the coefficients δ_1 and δ_2 satisfy in their multiplications the following rules

$$\delta_1^{2-} = \delta_2^2 = 1, \quad \delta_1 \times \delta_2 = 0.$$

It is easy to prove that the statistical mixture state described in Eq. (3) can be written in the form:

$$\begin{aligned} \rho_{SM}(0) &= \tfrac{1}{2} \left(|\alpha\rangle \, \langle\alpha| + |-\alpha\rangle \, \langle-\alpha|\right) \\ &= |\psi_{SM}(0)\rangle \, \langle\psi_{SM}(0)| \, . \end{aligned} \tag{6}$$

We must stress that the function $|\psi_{SM}(0)\rangle$ is not to be taken in the usual souse of the wave function, but must be treated as a functional tool to facilitate the calculations and treat the pure and statistical mixture states in the same footing.

For the superposition state Eq. (2) C_n in this case is just

$$C_n = \frac{1}{\sqrt{A}} \left[1 + (-1)^n r\right].$$

The solution of the Schrödinger equation in the interaction picture is given by

$$|\psi_I(t)\rangle = \sum_{n=0}^{\infty} C_n \{A_n(t) \, |n, e\rangle + B_n(t) \, |n+1, g\rangle\}, \tag{7}$$

with the coefficients $A_n(t)$ and $B_n(t)$ are

$$A_n(t) = q_n \exp[-i\lambda\gamma_n] \left\{\cos \lambda t F_n - i W_n \frac{\sin \lambda t F_n}{F_n}\right\} \tag{8}$$

$$B_n(t) = -i q_n v_n \exp[-i\lambda t \gamma_n] \frac{\sin \lambda t F_n}{F_n} \tag{9}$$

$$
\begin{aligned}
F_n^2 &= W_n^2 + v_n^2, \quad v_n = \sqrt{\frac{(n+k)!}{n!}}, \\
W_n &= \frac{\delta}{2} + \delta_-(n) + \eta_-(n) \\
\gamma_n &= \delta_+(n) + \eta_+(n) \\
\delta_\pm(n) &= \frac{n \pm R^2(n+k)}{2R}, \; R^2 = \beta_1/\beta_2, \; \delta = \Delta/\lambda \\
\eta_\pm(n) &= \frac{\chi n(n-1) \pm \chi(n+k)(n+k-1)}{2\lambda}
\end{aligned} \tag{10}
$$

With the wave function $|\psi_I(t)\rangle$ calculated, any property related to the atom or the field can be calculated. The reduced density matrix of the field (particle) of the system can be written as

$$
\rho_{f(a)}(t) = Tr_{a(f)}\{|\psi_I(t)\rangle \langle\psi_I(t)|\} \tag{11}
$$

The dynamics described by the Hamiltonian (1) leads to an entanglement between the field and the particle (atom or trapped ion). In order to calculate the partial entropies, we must obtain the eigenvalues of the reduced field density operator and the reduced particle density operator. Employing the reduced density operator for the particle or the field, we investigate the properties of the entropies (S_a, S_f) and entanglement in the framework of the multi-quanta JC model.

3. DEGREE OF ENTANGLEMENT

Despite the fact that the basic idea of quantum entanglement was acknowledged almost as soon as quantum theory was discovered, it is only in the last few years, that consideration has been given to finding mathematical methods to generally quantify entanglement (Bennett *et al.*, 1996b; Henderson and Vedral, 2000; Horodecki *et al.*, 2000). In the case of a pure quantum state of two subsystems, a number of widely accepted measures of entanglement are known. However, the question of quantifying the degree of entanglement for general mixed states is still under discussion (Bose *et al.*, 2001a,b).

In this article, we study the evolution of the partial entropy of the atom and field subsystem in the frameworks of the JC model. We will compare the behaviour of the system in the case of having a coherent superposition state *SS* and a statistical mixture of coherent states *SM* as an initial field with both the Stark shift and Kerr-like medium in multi-photons process. Let us now briefly repeat some of key underlying definitions. The entropy S of a quantum-mechanical system described by the density operator $\hat{\rho}$ is defined as follows:

$$S = -Tr\{\hat{\rho} ln \hat{\rho}\}, \quad (12)$$

where we have set the Boltzmann constant K equal to unity If $\hat{\rho}$ describes a pure state, then $S = 0$, and if $\hat{\rho}$ describes a mixed state, then $S \neq 0$. Entropies of the atomic and field sub-systems are defined by the corresponding reduced density operators:

$$S_{a(f)} = -Tr_{a(f)}\{\hat{\rho}_{a(f)} ln \rho_{a(f)}\}. \quad (13)$$

By using numerical calculations, we calculate the eigenvalues of the reduced density operator for the particle and the field and then calculate the entropies to investigate quantum entanglement in the composite system.

Let us now come to specific numerical examples to investigate the influence of superposition of coherent states of light on the evolution of the entropies and entanglement with the Stark shift and Kerr-like medium added in multi-quanta JC model.

To investigate the influence of the superposition state (*SS*) and statistical mixture state (*SM*) in the presence of the Stark shift and Kerr-like medium, we display entropy of the particle initially prepared in the excited state as functions of the scaled time λt with fixed initial mean number of quanta $\overline{n} = 25$ and different values of χ/λ (namely (0, 0.05 and 0.5 for $k = 1$) and (0, 0.01 and 0.5 for $k = 2$)) and with different values of Stark shift parameter (namely $R = 1(\beta_1 = \beta_2)$, $R = 0.3$ and $R = 0.5$).

3.1. One-photon Process

In Figs. 1–3, we display the entropies for one-photon process in the exact resonance, i.e., the detuning parameter $\Delta/2\lambda = 0$ with the fixed initial mean numbers of quanta $\bar{n} = 25$, when the field initially in a superposition state (*SS*) Eq. (2) ((*a*) $r = 0$ (coherent state), (*b*) $r = 1$ (even coherent state)) and (*c*) a statistical mixture states (*SM*) Eq. (3) with different values of χ/λ (namely 0, 0.05 and 0.5) respectively. In Figure 1, we see the influence of the initial field (*SS* or *SM*) on the entropy in the absence of Kerr-like medium, as seen from this figure the entropy in *SS* ($r = 0$ (coherent state), see Fig. 1(a)). It is observed that the maximum entropy is achieved for the evolution of (S_a, S_f) during the first stage of the time evolution, then the entropy evolves to minimum values and the field is completely disentangled from the particle at half of the atomic inversion revival time $\left(t_R = 2\pi\sqrt{\bar{n}}/\lambda = \frac{10\pi}{\lambda}\right)$. When time increases, the values of the minimum entropy increase. Finally, the entropies (S_a, S_f) remains maximum as time increases, these results are the same as in Ref. (Phoenix and Knight, 1988; Fang and Zhou, 1994a). Also it can been seen that the behaviors of the entropy are not periodical in the the absence of Kerr-like medium (Phoenix and Knight,

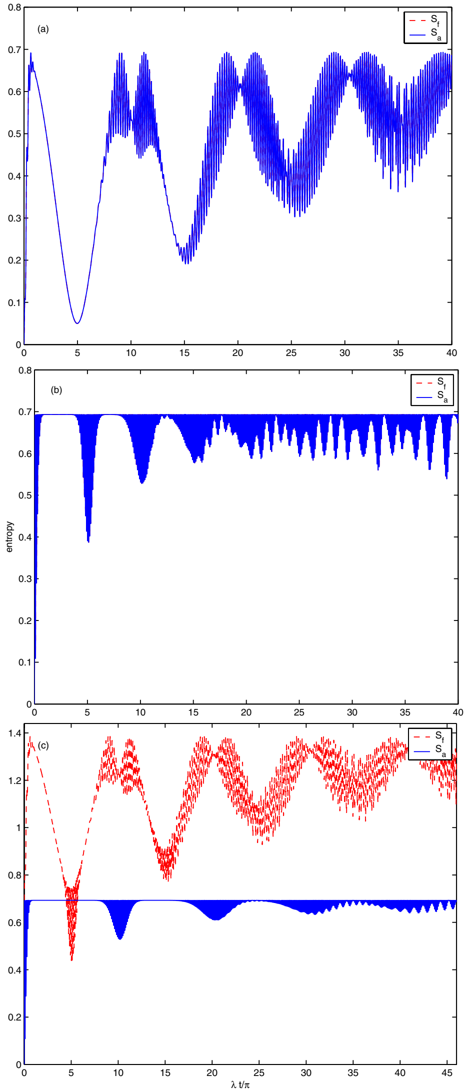

Fig. 1. The entropy for the atom S_a (solid curve) and the field entropy S_f (dashed curve) as a function of the scaled time λt of the particle initially prepared in the excited state and the field initially prepared in: (*a*) a superposition state *SS* ($r = 0$ (coherent state)), (*b*) a superposition state ($r = 1$ (even coherent state)) and (*c*) a statistical mixture (*SM*) of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ ($\overline{n} = 25$).

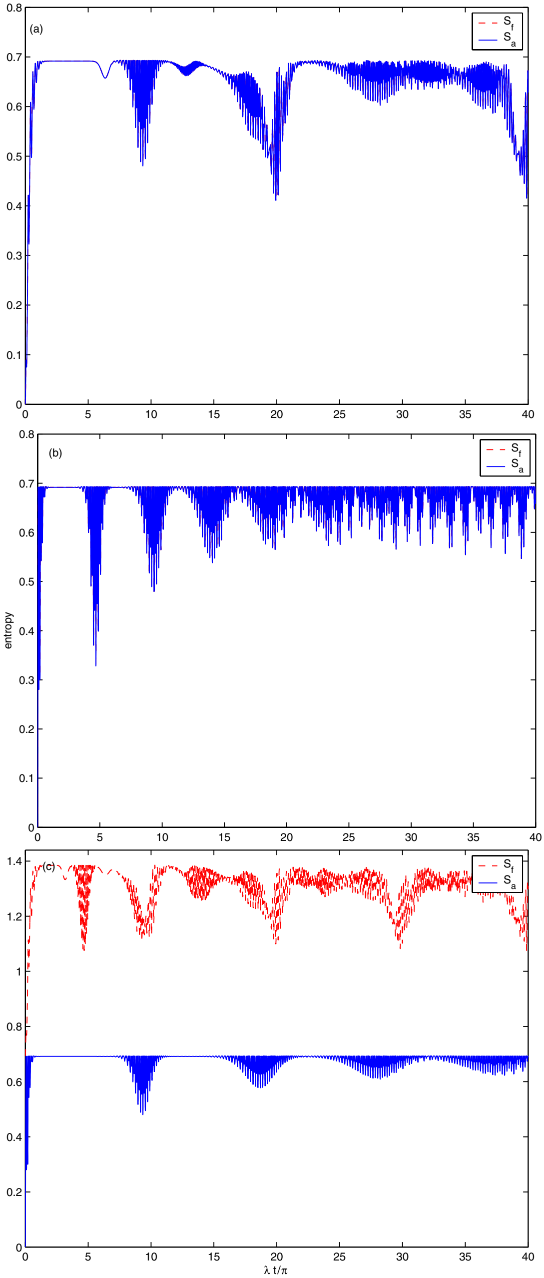

Fig. 2. The same as in figure (1) but with the Kerr-like medium parameter $\frac{\chi}{\lambda} = 0.05$.

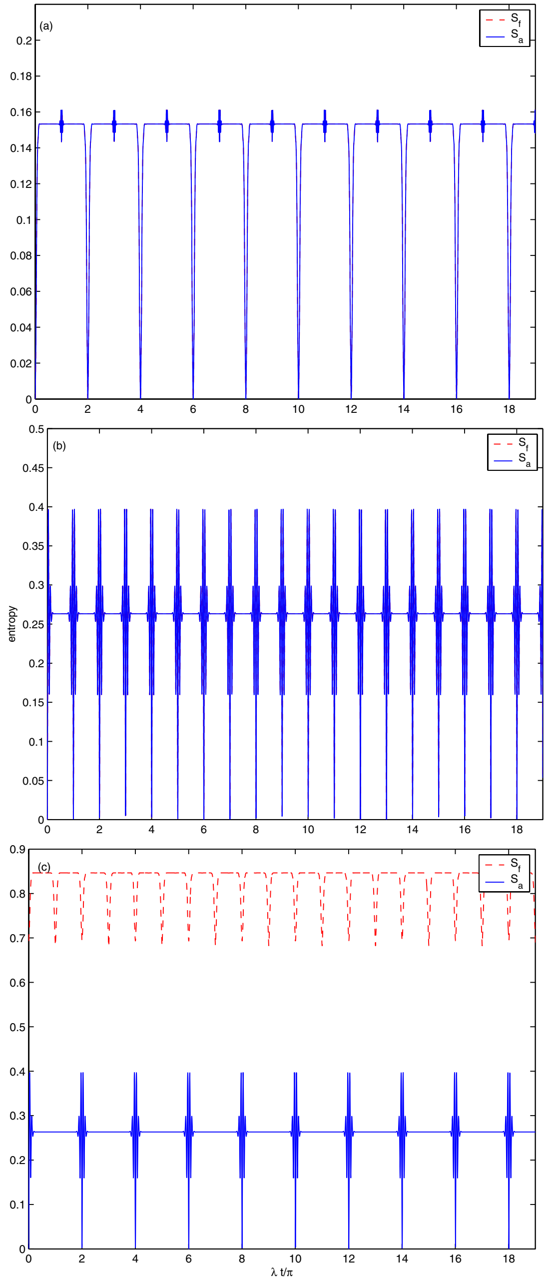

Fig. 3. The same as in figure (1) but with the Kerr-like medium parameter $\frac{\chi}{\lambda} = 0.5$.

1988; Fang and Zhou, 1994a). While in the superposition state *SS* ($r = 1$ (even coherent state)) (see Fig. 1(b)), we note that the entropy appears as the *SM* for the atom S_a, (see Fig. 1(c) solid curve), but it evolves to minimum at half of the revival time of *SM*. This effect is due to the interference between the two coherent states in the superposition state, and it can be understood looking at the photon number distribution of the initial fields.

Hence this signal gives a clear measure of the remaining degree of coherence between the two components of the cat state, while the signals present in both cases are due to intrinsic revivals of each component individually.

In Fig. 1(c), in a statistical mixture states (*SM*) the entropies (S_a and S_f) are difference from each other and from *SS* case, where in *SS* the entropies are the same (see Fig. 1(a) and (b)), but here in *SM* the entropy for the field S_f is greater than the entropy for the atom S_a (see Fig. 1(c)), the entropy for the atom is sustained stable at the maximum values at the half of a new revival time, while S_f evolves to minimum values and it resembles the coherent state case, but at higher values compare (Fig. 1(a)–(c)).

To visualize the influence of the Kerr-like medium on the entropy we set different values of χ/λ, and all the other parameters are the same as in Fig. 1 for one photon process. The outcome is presented in Figs. 2 and 3. As seen from these figures, that weak nonlinear interaction of the Kerr-like medium with field mode leads to increasing values of the minimum entropy and the sustainment time of maximum entropy (see Figs. 2 and 3). In this case, the field and the particle almost retain a strong entanglement in the time evolution process, but they become completely disentangled at certain values of time (almost at every $\lambda t = n\pi$). With the increase of the non-linear Kerr-like medium with field mode (e.g. $\chi/\lambda = 0.5$), the value of the maximum entropy decreases except for some instants (see Fig. 3). In this case the degree of entanglement between the field and the particle reduces. Finally, in the strong nonlinear interaction of the Kerr medium with the field mode, numerical studies leads to periodic evolution of the field entropy with periodicity π or 2π (see Fig. 3).

3.2. Two-photon Process

In Figures 4–8, we display the entropies for two-quanta ($k = 2$) in the exact resonance with the fixed initial mean numbers of quanta $\bar{n} = 25$ as above. Fig. 4 shows the influence of the initial field (*SS* or *SM*) on the entropies in the absence of the Stark shift and Kerr-like medium. As seen from these figures the entropies in *SS* ($r = 0$ (coherent state), see Fig. 4(a), it is observed that the entropy evolves with a period π/λ, when $t = n\pi/\lambda$, $n = 0, 1, 2, 1/4$), the entropies (S_a, S_f) evolves to the minimum values, and the field is completely disentangled from the particle. While when $\lambda t = \left(n + \frac{1}{2}\right)\pi$, $n = 0, 1, 2 \ldots$), the entropies (S_a, S_f) evolves to the maximum values, and the field is completely entangled with the particle.

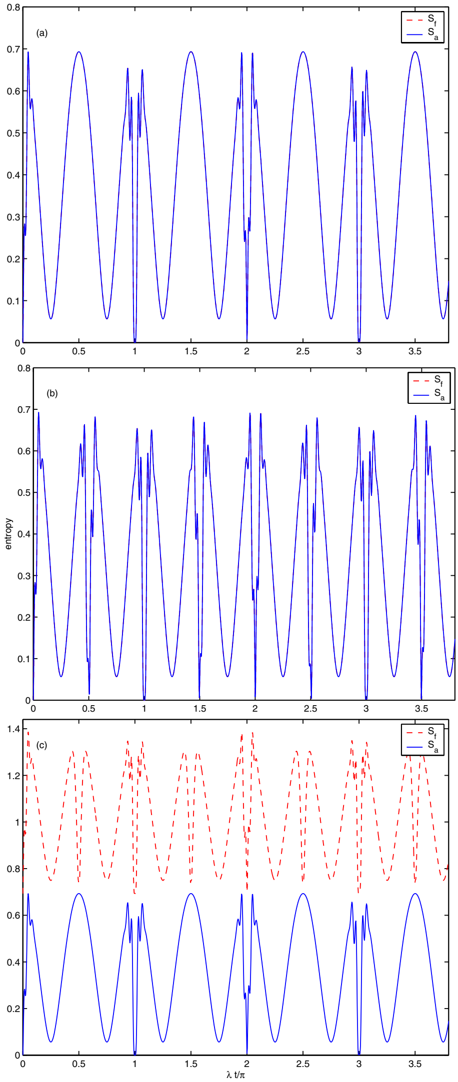

Fig. 4. The entropy for the atom S_a (solid curve) and the field entropy S_f (dashed curve) as a function of the scaled time λt of the particle initially prepared in the excited state and the field initially prepared in: (*a*) a superposition state *SS* ($r = 0$ (coherent state)), (*b*) a superposition state ($r = 1$ (even coherent state)) and (*c*) a statistical mixture (*SM*) of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ ($\overline{n} = 25$).

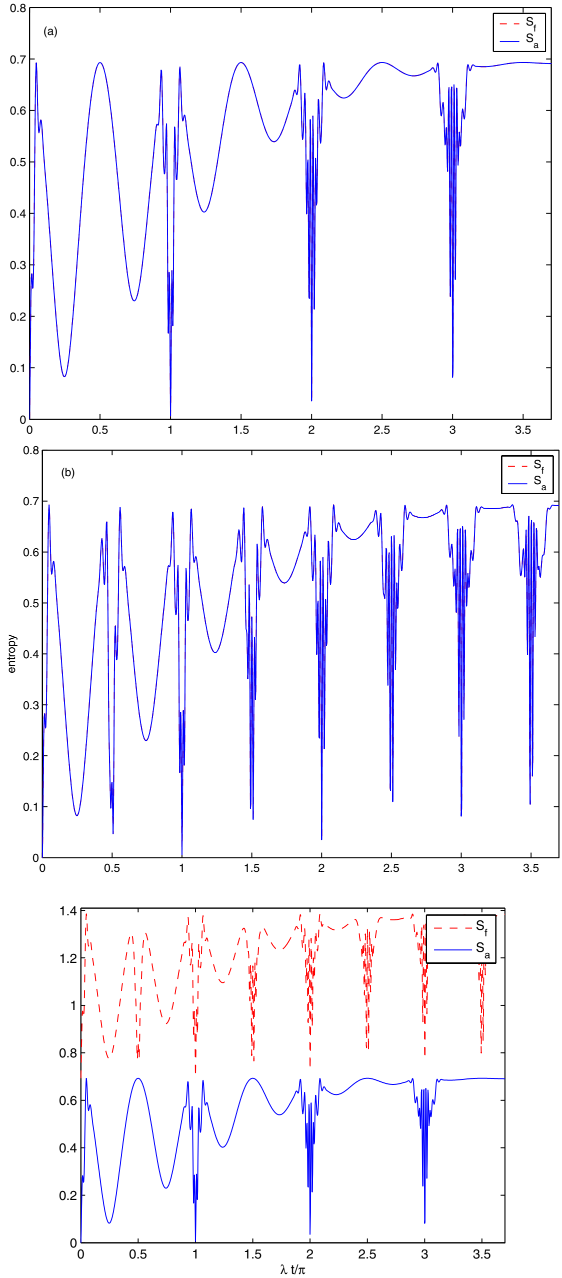

Fig. 5. The same as in figure (4) but with the Kerr-like medium parameter $\frac{\chi}{\lambda} = 0.01$.

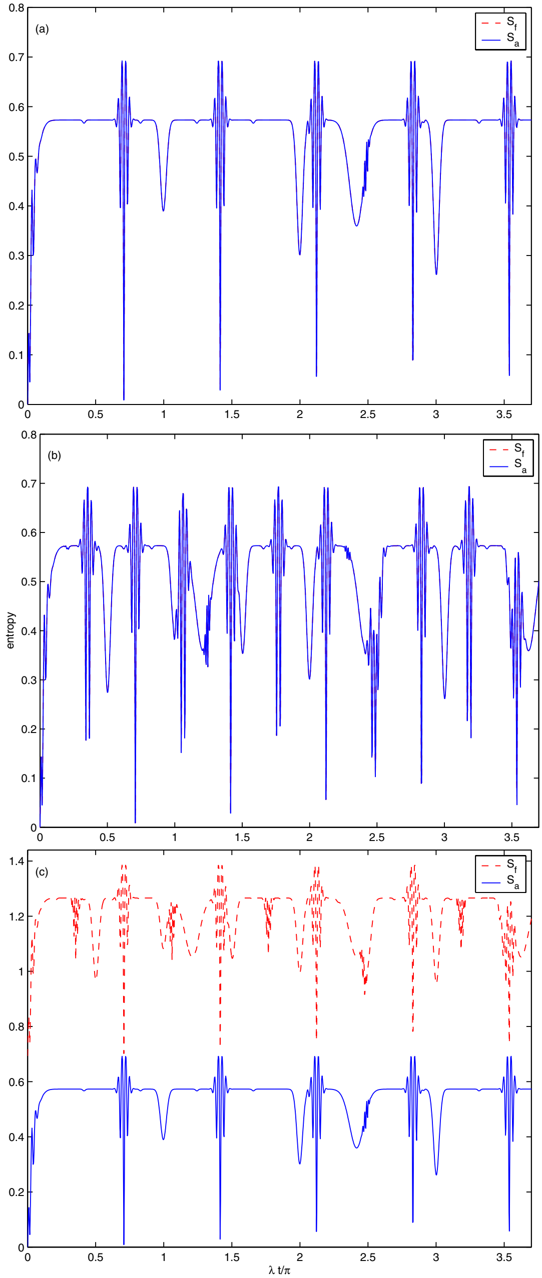

Fig. 6. The same as in figure (4) but with the Kerr-like medium parameter $\frac{\chi}{\lambda} = 0.5$.

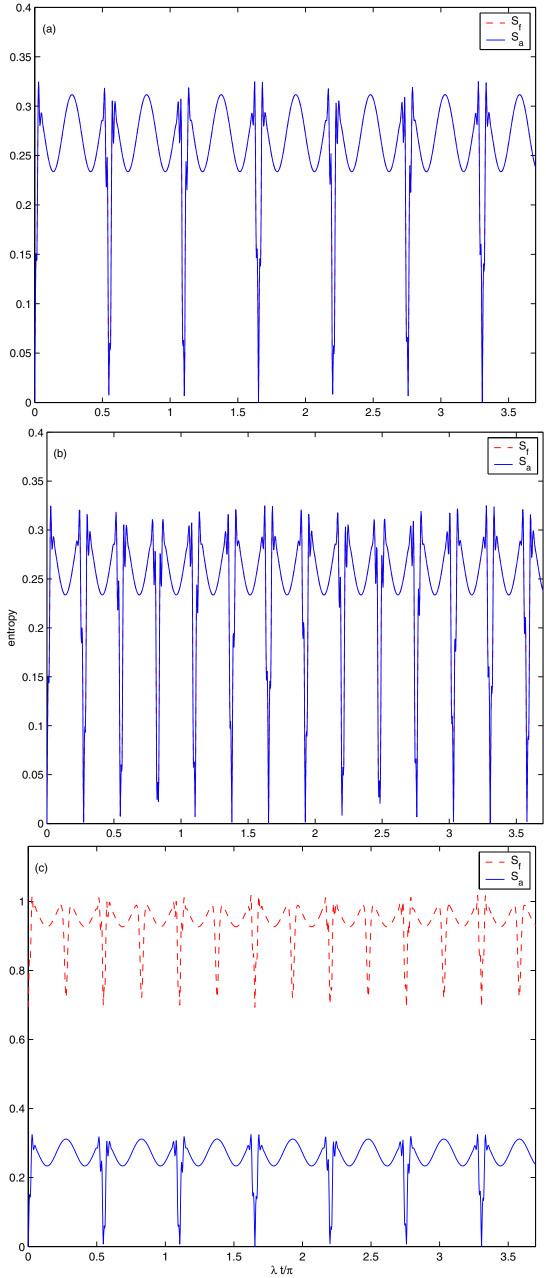

Fig. 7. The same as in figure (4) but with the Stark shift parameter $R = 0.3$.

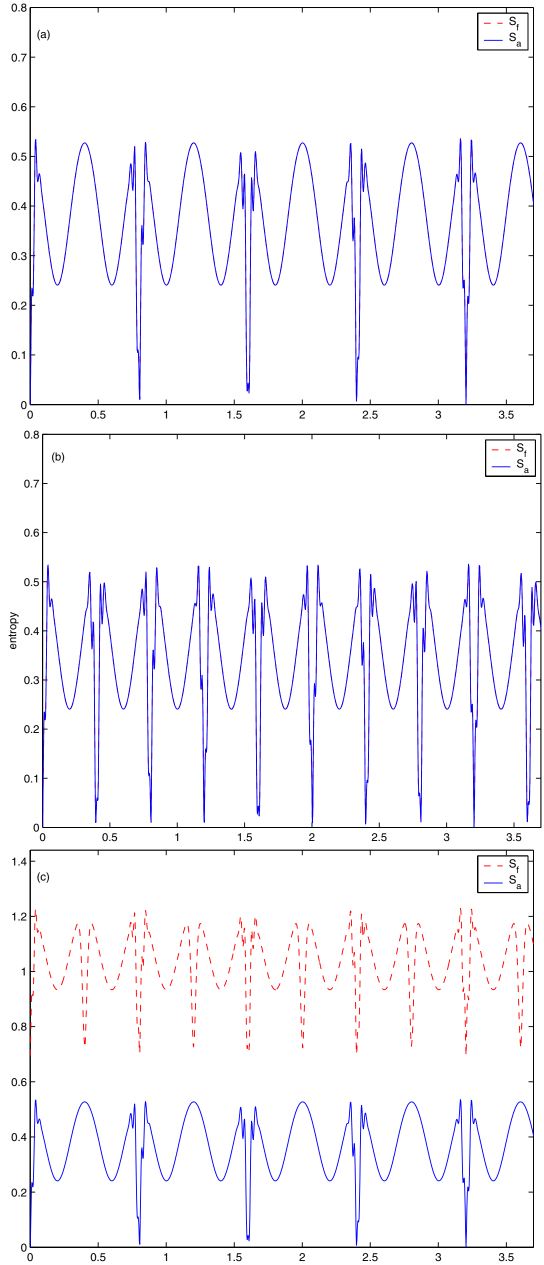

Fig. 8. The same as in figure (4) but with the Stark shift parameter $R = 0.5$.

While for the superposition state *SS* (even coherent state) see Fig. 4(b)), we note that for two-quanta process the entropy appear as the *SM* for the atom S_a, (see Fig. 4(c)), i. e., in a statistical mixture states (*SM*) the entropies (S_a, S_f) are very different from each other and from *SS*, where in *SS* the entropies are the same (see Fig. 4(a) and (b)). But here in *SM* the entropy for the field S_f is greater than the entropy for the atom S_a (see Fig. 4(c). The entropy for the atom is sustained stable at the maximum values at the half of the revival time, while S_f evolves to minimum values.

To visualize the influence of the Kerr-like medium on the entropies we set different values of χ/λ, and all the other parameters are the same as in Fig. 4. The outcome is presented in Figs. 5 and 6. As seen from these figures, the case when the nonlinear interaction of the Kerr-medium with the field mode is very weak (e.g. $\chi/\lambda = 0.05$) leads to increasing values of the minimum entropy and of the sustainment time of the maximum entropy.

With the increase of the nonlinear Kerr-like medium effect (e.g. $\chi/\lambda = 0.5$), the value of the maximum entropy begins to decrease except for some instants (see Fig. 6). In this case, the degree of entanglement between the field and the particle reduces. Finally, in the strong nonlinear interaction of the Kerr medium with the field mode limit for $\chi/\lambda > 1.0$ numerical studies show that the entropy tends almost to zero overall the time interval and the field can be a sustained in a pure state in this case. This result corresponds with the fact that in the strong nonlinear interaction of the Kerr medium with the field mode limit, the field and the atom are almost decoupled and the time evolution of the field is governed by the Hamiltonian $H_{eff} = \chi \hat{a}^{\dagger 2} \hat{a}^2$, which preserves zero value for the entropy.

To visualize the influence of the Stark shift on the entropies we set different values of Stark shift parameter, and all the other parameters are the same as in Fig. 4. The outcome is presented in Figs. 7 and 8, As seen from these figures, we show that the Stark shift leads to decreasing values of the maximum entropy and the evolution period of the entropy decrees with Stark shift parameter R (see Fig. 7, for $R = 0.3$ and Fig. 8, for $R = 0.5$). The more the parameters R is far from unity, the more the entropy is decreased.

4. CONCLUSIONS

We study the time evolution of the entropies and entanglement in the mixed state for multi-quanta JC model with an additional both the Stark shift and Kerr-like medium. By using a numerical method we calculate the partial entropy of the atom and field subsystem. We compare the behaviour of the system in the case of having a superposition of coherent states and a statistical mixture of coherent states as initial field states. We conclude that, in the *SS* ($r = 0$ (coherent state)), the field (atom) entropy reaches its minimum values at half of the revival time and the field and the particle are almost disentangled, while in the *SS* ($r = 1$ (even coherent state)), the entropies at the same time evolves to the minimum values and the field and the particle are strongly disentangled. Also in the statistical mixture state (*SM*), the entropy for the atom is sustained at maximum values at half of the revival time, while S_f evolves to minimum values. When we take the Stark shift into account, it leads to decrease the values of the maximum entropy and the evolution period of the entropy decreases with Stark shift parameter R. When the nonlinear interaction of the Kerr medium with the field mode is very weak, it leads to an increase of the sustainment time of the maximum entropy and weakens entanglement of the field with the particle. While when it is very strong, it results in almost zero value of the entropy and the field is disentangled from the particle during the time evolution. The effect of the *SM* states is shown in the difference between the atomic and the field entropies where it is shown in the figures that S_f is always greater than S_a.

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